Problem 1A.4

Gas-mixture viscosities at low density. The following data[2] are available for the viscosities of mixtures of hydrogen and Freon-12 (dichlorodifluoromethane) at 25°C and 1 atm:

Mole fraction of H_2:	0.00	0.25	0.50	0.75	1.00
$\mu \times 10^6$ (poise):	124.0	128.1	131.9	135.1	88.4

Use the viscosities of the pure components to calculate the viscosities at the three intermediate compositions by means of Eqs. 1.4-15 and 16.

Sample answer: At 0.5, $\mu = 0.01317$ cp.

Solution[1]

Eqs. 1.4-15 and 16 in the text are a pair of formulas that allow us to compute the viscosity of a mixture with N chemical species.

$$\mu_{\text{mix}} = \sum_{\alpha=1}^{N} \frac{x_\alpha \mu_\alpha}{\displaystyle\sum_{\beta=1}^{N} x_\beta \Phi_{\alpha\beta}}, \tag{1.4-15}$$

where $\Phi_{\alpha\beta}$ is

$$\Phi_{\alpha\beta} = \frac{1}{\sqrt{8}} \left(1 + \frac{M_\alpha}{M_\beta}\right)^{-1/2} \left[1 + \left(\frac{\mu_\alpha}{\mu_\beta}\right)^{1/2} \left(\frac{M_\beta}{M_\alpha}\right)^{1/4}\right]^2. \tag{1.4-16}$$

The gas mixture has two chemical species, dichlorodifluoromethane (CCl_2F_2) and hydrogen (H_2).

Species 1: CCl_2F_2

$$M_1 = 12.01 + 2(35.45) + 2(19.00) = 120.91 \ \frac{\text{g}}{\text{mol}}$$

$$\mu_1 = 124.0 \times 10^{-6} \text{ poise}$$

Species 2: H_2

$$M_2 = 2(1.008) = 2.016 \ \frac{\text{g}}{\text{mol}}$$

$$\mu_2 = 88.4 \times 10^{-6} \text{ poise}$$

As a result, Eq. 1.4-15 becomes

$$\begin{aligned}
\mu_{\text{mix}} &= \sum_{\alpha=1}^{2} \frac{x_\alpha \mu_\alpha}{\displaystyle\sum_{\beta=1}^{2} x_\beta \Phi_{\alpha\beta}} \\
&= \frac{x_1 \mu_1}{\displaystyle\sum_{\beta=1}^{2} x_\beta \Phi_{1\beta}} + \frac{x_2 \mu_2}{\displaystyle\sum_{\beta=1}^{2} x_\beta \Phi_{2\beta}} \\
&= \frac{x_1 \mu_1}{x_1 \Phi_{11} + x_2 \Phi_{12}} + \frac{x_2 \mu_2}{x_1 \Phi_{21} + x_2 \Phi_{22}}.
\end{aligned}$$

[1]Special thanks to GESE on Twitter for pointing out my mistake.
[2]J. W. Buddenberg and C. R. Wilke, *Ind. Eng. Chem.* **41**, 1345–1347 (1949).

Use Eq. 1.4-16 to determine Φ_{11}, Φ_{12}, Φ_{21}, and Φ_{22}.

$$\Phi_{11} = \frac{1}{\sqrt{8}}\left(1+\frac{M_1}{M_1}\right)^{-1/2}\left[1+\left(\frac{\mu_1}{\mu_1}\right)^{1/2}\left(\frac{M_1}{M_1}\right)^{1/4}\right]^2 = 1$$

$$\Phi_{12} = \frac{1}{\sqrt{8}}\left(1+\frac{M_1}{M_2}\right)^{-1/2}\left[1+\left(\frac{\mu_1}{\mu_2}\right)^{1/2}\left(\frac{M_2}{M_1}\right)^{1/4}\right]^2 \approx 0.092017$$

$$\Phi_{21} = \frac{1}{\sqrt{8}}\left(1+\frac{M_2}{M_1}\right)^{-1/2}\left[1+\left(\frac{\mu_2}{\mu_1}\right)^{1/2}\left(\frac{M_1}{M_2}\right)^{1/4}\right]^2 \approx 3.93432$$

$$\Phi_{22} = \frac{1}{\sqrt{8}}\left(1+\frac{M_2}{M_2}\right)^{-1/2}\left[1+\left(\frac{\mu_2}{\mu_2}\right)^{1/2}\left(\frac{M_2}{M_2}\right)^{1/4}\right]^2 = 1$$

μ_{mix} has to be calculated for three situations: (1) when $x_1 = 0.75$ and $x_2 = 0.25$,

$$\mu_{\text{mix}} = \frac{(0.75)\mu_1}{(0.75)\Phi_{11} + (0.25)\Phi_{12}} + \frac{(0.25)\mu_2}{(0.75)\Phi_{21} + (0.25)\Phi_{22}} \approx 127.2 \times 10^{-6} \text{ poise}$$

(2) when $x_1 = 0.50$ and $x_2 = 0.50$,

$$\mu_{\text{mix}} = \frac{(0.50)\mu_1}{(0.50)\Phi_{11} + (0.50)\Phi_{12}} + \frac{(0.50)\mu_2}{(0.50)\Phi_{21} + (0.50)\Phi_{22}} \approx 131.5 \times 10^{-6} \text{ poise}$$

and (3) when $x_1 = 0.25$ and $x_2 = 0.75$.

$$\mu_{\text{mix}} = \frac{(0.25)\mu_1}{(0.25)\Phi_{11} + (0.75)\Phi_{12}} + \frac{(0.75)\mu_2}{(0.25)\Phi_{21} + (0.75)\Phi_{22}} \approx 135.4 \times 10^{-6} \text{ poise}$$

Therefore,

Mole fraction of CCl_2F_2 (x_1):	0.75	0.50	0.25
Mole fraction of H_2 (x_2):	0.25	0.50	0.75
$\mu_{\text{mix}} \times 10^6$ (poise):	127.2	131.5	135.4
$\mu \times 10^6$ (poise):	128.1	131.9	135.1

These numbers are in excellent agreement with the data.